Journal of Islamic Monetary Economics and Finance, Vol.2, No.2 (2017), pp.173-192
p-ISSN: 2460-6146, e-ISSN: 2460-6618

A MATHEMATICAL MODEL OF PROFIT-LOSS SHARING SCHEME OF SMALL INVESTMENT FOR TRADITIONAL MARKET TRADERS USING THE SEMI-FUZZY LOGIC APPROACH

Novriana Sumarti [1]
Adythia Dean Marendri [2]

Abstract

A mathematical model of micro-finance investment using profit-loss sharing scheme are made and implemented to simulated data. Here profits from the venture will be shared in a portion between the investor and the entity running the business. The scheme can be classified as *Musharaka* type of investment in Sharia economy. The proposed model is theoretically implemented with data from small-scale traders at a local traditional market who have small turnover. They are common target of usurers who lend money with high interest rate and penalties. If the traders are in unfortunate conditions, they are potentially in poorer condition than before committing themselves to the usurer. In the conventional practices of the profit sharing scheme, the investor will get a fixed portion of the trader's income, which is applied for all kind of small-scale traders. If the traders are diligent and hard worker and have very high turnover, then the investor will gain much more profit whether the contributed capital is small or large. In this paper, the scheme is implemented using Semi-Fuzzy Logic Approach so that the profit-loss sharing scheme can achieve its intended goal, which is to make a profitable investment not only for the investor but also for traders. The approach is not fully using Fuzzy Logic because some variables are still in crisp numbers and the optimization problem is regular in the form of crisp numbers. Based on the existing data, the results show that the optimal profit share is depended on the income of the traders. The higher the income coming from the venture, the lesser the profit share for the investor which is reasonable with the fixed initial contributed capital.

Keywords: Profit-Loss Sharing, Fuzzy logic, Musharaka, Optimization, Mathematical model

JEL Classification: C61, D60

Received: October 2, 2016; Revised: January 13, 2017;
Accepted: January 30, 2017

---

1 Industrial and Financial Mathematics Research Group, Institut Teknologi Bandung, Indonesia. email: novriana@math.itb.ac.id (Corresponding author)
2 Industrial and Financial Mathematics Research Group, Institut Teknologi Bandung, Indonesia. email: adythia.96.01@gmail.com

I. INTRODUCTION

Sharia Economy is the economic system regulated by the Al-Quran and Al-Hadith. There are types of contract (*Akad*), which are agreements between two or more parties to do or not do certain legal actions. The classification of contracts are based on the objective, validity and implementation of the contract. For instance, the classification of economic activities based on the objective are either non-profit (charitable) or profit activities. The contract of borrowing money is a charity and should not be a profit activity. According to Al-Hadith (Ensiklopedia Ayat Al-Qur'an & Hadits, 2016), the exchange of gold with gold or silver with silver should be in the same amount and there should not be an increase, except they are of different types. It also applies to money because nowadays money has the same functionality with gold/silver as an exchange tool. Does this mean that Islam opposed to investment, in which a person lends money to a businessman to do economic activities in order to get shared profit? In the contrary, there are Sharia rules on investment through cooperation contracts with profit sharing. For example, *Mudarabah* is an investment contract between a capital owner (investor) and an entrepreneur, where the capital provided entirely by the investor. The investment profits are shared in a certain ratio (*nisbah*) between two parties, while losses is borne only by the investor. Another example is the *Musharaka*, where ownerships of the capital are held by the investor and the entrepreneur, so that the wealth of the company is indistinguishable. Profits and losses are shared in certain ratio for both sides. It should be emphasized that the ratio is respect to the total gains of the investment, but not to the amount of fund initially deposited by the investor.

In this paper, a mathematical model of micro-finance investment, that can be classified as *Musharaka*, using profit-loss sharing scheme are made and implemented to simulated data. The parties involved are an investor and entities running the business, which are small-scale traders at a local traditional market who have small turnover.

II. LITERATURE REVIEW

This The fundamental difference between Islamic and conventional banking lies in the principals involved in the financial transactions and operations. The financial transactions are including profit loss sharing (PLS) contracts such as *Mudarabah* and *Musharaka*. One of the main characteristics of Islamic banking is that the assets are concentrated on asset-based investments that has a credit risk but also it is also backed by a real asset (Greuning & Iqbal, 2009). As the result, the lending capacity of the Islamic banking sector (at least for commercial banks) is bound by the availability of real assets in the economy. In consequence, Islamic financial institutions put more emphasis on improving productivity. Eventually the growth of Islamic banking will give very significant contribution to economic performance and growth. Poverty and unemployment will directly resolved through the good economic performance.

In (Kumru & Sarntisart, 2016), the growth and welfare implications of Islamic banking systems are observed by developing a theoretical model with a case where a sizeable portion of Muslims do not trust conventional banks. The numerical analysis result shows that in an economy populated with a certain number of religiously concerned individuals, the existence of an Islamic banking system can generate higher growth and improve welfare substantially. Islamic banks can foster economic growth and improve welfare.

Authors in (Imam & Kpodar (2016) learned that most of the countries with large Islamic populations are often low-income population so they have underdeveloped financial systems. Islamic banking has unique characteristics that appears better adapted to characteristics prevailing in poorer countries of the Middle East, sub-Saharan Africa, and Asia. The paper wants to assess whether the development of Islamic banking is good for economic growth in 52 countries with data covering the period 1990-2010. They pointed out some weakness of Islamic Banking; First is lack of economies of scale, where they are often newer and smaller than conventional banks, so they are often still below optimal scales and thus have higher cost structures. Secondly, there is lack of liquid instruments, where there is no secondary market for Islamic fixed-income products, forcing Islamic banks to have large liquidity buffers, putting them at a disadvantage relative to conventional banks.

There are some indicators to measure Islamic banking development, those are the credit extended by these banks to the private sector divided by nominal GDP, the ratio of Islamic banking assets to GDP and the ratio of deposits in Islamic banks to GDP. The indicators of the economy growths are initial real GDP per capita, inflation, government consumption, education, trade openness, and terms of trade. Using robust empirical techniques, they claim countries with developing Islamic banking have experienced faster economic growth than others, by assuming that the level of financial deve-lopment and other growth determinants are held as constant. Furthermore, adopting financial regulations of Islamic banking by non-Islamic countries can re-energize the growth.

Authors in (Beck, Demirgüç-Kunt, & Merrouche, 2013) show few significant differences in business orientation between Islamic and conventional banks. Islamic banks are less cost-effective, but have a higher intermediation ratio, higher asset quality and are better capitalized. Islamic banks are better capitalized, have higher asset quality and are less likely to disintermediate during crises. The better stock performance of listed Islamic banks during the recent crisis is also due to their higher capitalization and better asset quality.

In (Hamza, 2016), the investment deposits, that are consi-dered in the Islamic bank as a form of limited-duration equity investment, are loss absorbent capital under PLS principle. The return of the investment deposit will vary with assets performance and therefore it is not guaranteed by the bank. On the other hand, the bank is facing hard competition from conventional banks offering guaranteed term deposit and where competition envi-ronment is often favourable to conventional financing activities. The author examined the reaction of investment deposit return to the effect of excessive risk taking, competition environment and governance structure. A pooled regression model was applied to a panel of sixty Islamic banks from 15 countries during the period 2004-2012. The result show that the manage-ment of investment deposit and PLS assets is characterized by a moral hazard behaviour and excessive risk taking. It seems that capital ratio and interest rate affect positively to the investment deposit return. Small Islamic banks in the data sample offer a better return of deposit compared to the large bank.

Generally, it seem that small banks or microfinance institutions could be a solution in the developing economy of countries. Microfinance is the provision of financial services to low income individuals who lack access to the conventional banking sector. The early stage microfinance institutions (MFIs) were non-profit organisations with a mission to alleviate poverty by helping the poor develop vocational and business management skills, and by giving them small, uncollateralised loans, mostly to be used as working capital. In (Yunus, 1999) and (Haque, 2000), the well-known Grameen Bank, located in Bangladesh, had shown the role of micro-lending mechanisms in creating sustainable development, by raising the standard of living among its clients and, in return, alleviating the poverty cycles. In (Amran, Rahman, Yusof, & Mohamed, 2014), the current practices of Islamic Microfinance Institutions (IMFIs) in Malaysia are discussed in providing *Qard Hasan*, that is the loan for promoting trade activities in terms of entrepreneurship development among the microfinance borrowers. Having equipped with the application of Mobile Banking to widen the access of financial services, it claims that IMFIs are the successful model.

The mathematical model of the investment on small enterprises or Microfinance using the profit-loss sharing (PLS) scheme for *Musharaka* type of investment had been developed in previous studies (Sumarti, Athari, & Fadhli, 2008), (Sumarti, Fitriyani, & Damayanti, 2014) and (Sumarti, Sidarto, Syamsuddin, Mardiyyah, & Rizal, 2015). Mathematical modeling of PLS schemes has not been studied frequently elsewhere. In (Ahmed, 2002), the model is still a simple equation, where financial theories are not considered. The model is improved in (Abu-Joudeh, 2012), but it is still a simple one and not easily applied. One important question in PLS is how to determine a fair portion (nisbah) in profit sharing between the investor and the entrepreneur. Given a constant capital A contributed by the investor, the entrepreneur, who is small-income traders at a traditional market, will pay back the capital plus the shared profit daily. If the portion is fixed to all type entrepreneurs, whether they have potentially big or small return of their investment, the shared profit from a hard worker trader will be much larger than the amount of the contributed capital from the investor. This seems unfair to the entrepreneur. This study has purpose to find the fair portion in PLS scheme for the investor and entrepreneur.

The developed mathematical model is implemented on sets of real data in order to validate the purpose. The real data were simulated by Monte-Carlo method in previous studies. Firstly the original data are identified their distribution. New sets of data particular on the identified distributions are generated repeatedly in order to see the general performance of the model on that kind of data. In this paper we use different approach by the fuzzy logic, so the data is clustered to identify different kinds of data. In sequence, the model is implemented on those clusters. So we does not need to generate simulation data in order to provide different kinds of conditions. It will be shown that although the approach is different, the results will be similar and support the previous result. We expect that the model is appropriate for solving problem in determining the optimal portion share in the profit-loss sharing scheme.

The use of fuzzy logic in the decision-making process is inevitable in various fields, due to the need for modelling the uncertainty and vagueness. One of the advantage is the determination of fuzzy rules with linguistic terms, which are easily to understand (Yu, Wang, & Chen, 2006). The implementation of fuzzy logic is including in economy and finance. Some researches in economy is, for instance, the modelling of under-ground economy in a particular country in (Yu *et al.*, 2006), investment analysis in (Sumarti & Sukaenah, 2010) and (Sumarti & Dhanny, 2010), and others. In finance, the fuzzy logic has been implemented in portfolio selection problem, such as in (Hasuike, Katagiri, & Ishii, 2009), and (Sumarti & Wahyudi, 2014), option pricing in (Guerra, Sorini, & Stefanini, 2011), (Chrysafis & Papadopoulos, 2009), and (Sumarti & Nadya, 2016), and others.

In the next section the definitions and some arithmetic operations of fuzzy sets are explained. The mathematical modelling of profit-loss sharing is explained in section 3.2 in the original form (Sumarti *et al.*, 2014) and (Sumarti *et al.*, 2015), and then is transformed to the new form in the framework of fuzzy sets in section 3.3. The transformation from non-fuzzy framework to fuzzy sets framework, for solving the optimization problem stated from the mathematics modelling, is quite challenging especially finding feasible solutions that satisfy the constraints, such as in section 4. Some operations are newly defined especially in comparing the numerical value of two fuzzy numbers. In section 5, the results are interpreted so it can show the model gives good performance and is reaching its goal.

III. METHODOLOGY

3.1 Fuzzy Logic

Fuzzy logic has many valued logic in which the truth values of variables may be any real number between 0 and 1. The truth values are described as the degree of membership of an element in the fuzzy set whose value lies in the interval [0,1]. Let $\bar{A}$ be a set called a fuzzy number. The degree of the membership function $\mu_{\bar{A}}$ of an element x in $\bar{A}$ is defined as follows:

If $\mu_{\bar{A}}(x) = 1$ then x is an exact member of $\bar{A}$

If $\mu_{\bar{A}}(x) = 0$ then x is not the member of $\bar{A}$.

If $\mu_{\bar{A}}(x) = \rho$ with $0 \leq \rho \leq 1$ then x is a member of $\bar{A}$ with membership degree ρ.

The types of membership functions are various. The triangular form $\bar{F} = [f^L, f^C, f^R]$ and the trapezoidal form $\bar{G} = [g^L, g^{Cl}, g^{Cr}, g^R]$ are respectively defined below.

$$\mu_{\bar{F}}(x) = \begin{cases} 0, & x \leq f^L \text{ and } x \geq f^R \\ \frac{x-f^L}{f^C-f^L}, & f^L < x < f^C, \\ 1, & x = f^C, \\ \frac{f^R-x}{f^R-f^C}, & f^C < x < f^R. \end{cases} \tag{1a}$$

$$\mu_{\bar{G}}(x) = \begin{cases} 0, & x \leq g^L \text{ and } x \geq g^R \\ \frac{x-g^L}{g^{Cl}-g^L}, & g^L < x < g^{Cl}, \\ 1, & g^{Cl} \leq x \leq g^{Cr}, \\ \frac{g^R-x}{g^R-g^{Cr}}, & g^{Cr} < x < g^R. \end{cases} \tag{1b}$$

Note that non-fuzzy or crisp numbers can be defined with a membership function that have value either 1 or 0 only.

Let $\bar{A} = [a^L, a^{Cl}, a^{Cr}, a^R]$ and $\bar{B} = [b^L, b^{Cl}, b^{Cr}, b^R]$ be fuzzy numbers of the trapezoidal form, k is Real number, where their membership functions are defined as below.

$$\mu_{\bar{A}}(x) = \max\left(\min\left(\frac{x - a^L}{a^{Cl} - a^L}, 1, \frac{a^R - x}{a^R - a^{Cr}}\right), 0\right),$$

$$\mu_{\bar{B}}(y) = \max\left(\min\left(\frac{y - b^L}{b^{Cl} - b^L}, 1, \frac{b^R - y}{b^R - b^{Cr}}\right), 0\right).$$

Based on (Banerjee & Roy, 2012), arithmetic operations between two fuzzy numbers or between fuzzy and non-fuzzy numbers, resulted as $\bar{C} = [c^L, c^{Cl}, c^{Cr}, c^R]$, are defined below.

- $\bar{A} + \bar{B} = \bar{C},\ \bar{C} = [a^L + b^L, a^{Cl} + b^{Cl}, a^{Cr} + b^{Cr}, a^R + b^R]$,

  $\mu_{\bar{C}}(z) = \sup\left(\min\left(\mu_{\bar{A}}(x), \mu_{\bar{B}}(y)\right) : x + y = z\right)$,

which means:

$\mu_{\bar{C}}(z) = \sup\left(\min\left(\frac{x-a^L}{a^{Cl}-a^L}, \frac{y-b^L}{b^{Cl}-b^L}\right) : x + y = z\right)$ if

$a^L \leq x \leq a^{Cl}, b^L \leq y \leq b^{Cl}$, or

$\mu_{\bar{C}}(z) = 1$ if $a^{Cl} \leq x \leq a^{Cr}, b^{Cl} \leq y \leq b^{Cr}$, or

$\mu_{\bar{C}}(z) = \sup\left(\min\left(\frac{a^R-x}{a^R-a^{Cr}}, \frac{b^R-y}{b^R-b^{Cr}}\right) : x + y = z\right)$ if

$a^{Cr} \leq x \leq a^R, b^{Cr} \leq y \leq b^R$.

- $\bar{A} - \bar{B} = \bar{C},\ \bar{C} = [a^L - b^R, a^{Cl} - b^{Cr}, a^{Cr} - b^{Cl}, a^R - b^L]$,

  $\mu_{\bar{C}}(z) = \sup\left(\min\left(\mu_{\bar{A}}(x), \mu_{\bar{B}}(y)\right) : x - y = z\right)$, which means

  $\mu_{\bar{C}}(z) = \sup\left(\min\left(\frac{x-a^L}{a^{Cl}-a^L}, \frac{y-b^L}{b^{Cl}-b^L}\right) : x - y = z\right)$ if

  $a^L \leq x \leq a^{Cl}, b^L \leq y \leq b^{Cl}$, or

$\mu_{\bar{C}}(z) = 1$ if $a^{Cl} \leq x \leq a^{Cr}, b^{Cl} \leq y \leq b^{Cr}$, or

$\mu_{\bar{C}}(z) = \sup\left(\min\left(\frac{a^R-x}{a^R-a^{Cr}}, \frac{b^R-y}{b^R-b^{Cr}}\right): x - y = z\right)$ if

$a^{Cr} \leq x \leq a^R, b^{Cr} \leq y \leq b^R$.

- $k \odot \bar{A} = \bar{C},\ \bar{C} = [k \odot a^L, k \odot a^{Cl}, k \odot a^{Cr}, k \odot a^R]$, where operator $\odot$ can be replaced by any operator $+, -, \times, \div$.

$\mu_{\bar{C}}(z) = \sup(\max(\mu_{\bar{A}}(x)) : k \odot x = z)$ which means

$\mu_{\bar{C}}(z) = \sup\left(\min\left(\frac{x-a^L}{a^{Cl}-a^L}\right): k \odot x = z\right)$ if $a^L \leq x \leq a^{Cl}$, or

$\mu_{\bar{C}}(z) = 1$ if $a^{Cl} \leq x \leq a^{Cr}$, or

$\mu_{\bar{C}}(z) = \sup\left(\min\left(\frac{a^R-x}{a^R-a^{Cr}}\right): k \odot x = z\right)$ if $a^{Cr} \leq x \leq a^R$.

The latest operation defines a pointwise operation between a crisp number and a fuzzy number. This new definition is needed in this paper due to all parameters and in the model cannot always be transformed to fuzzy number.

3.2 Mathematical Modelling of the Scheme of Profit-Loss Sharing

The mathematical model for a profit-loss sharing (PLS) scheme is formulated in order to see how this scheme can replace the traditional practice of lending money against high interest by usurers. It could be implemented for small-scale investments of traditional-market traders. In this model, the investor gives a fixed capital A to a trader who has trading business for a quite long time. The trader should give money back daily as the return of the investment for a period of T days. The procedure of daily payments is proposed in (Yunus, 1999) and, additionally, the traders are deliberately assumed to be women, (Yunus, 1999) and (Sumarti *et al.*, 2008). The basic instalment is $I^* = \frac{A}{T}$. We have the real data collection $\omega_t, t = 1, \ldots, T$ which are the profits of daily trading of the trader, and its average $\bar{\omega}$. The total amount of payment $S_t(p)$ at day *t* consists of the basic instalment for repaying the money and a portion of profit sharing, written as follows:

$$S_t(p) = I_t + B_t(p) + C_t\ ,\ t = 1,2,\ldots,T. \quad (2)$$

I_t is the instalment at day *t* for paying the principal. The sharing of profit is determined in variable $B_t(p)$, which contains profit sharing portion *p*. C_t is the payable debt at day *t*, which will be explained later.

The loss sharing will be reflected in the values of all three variables. If the trader suffers a loss (which will be defined later), she is exempt from paying $B_t(p)$ and I_t on that day. However, she still has to pay the basic instalment I^* later, which is accumulated in the debt, H_t. Remember that the term debt is used for H_t due to late payment of the basic instalment. There is no penalty for this late payment. Detailed formulation about the basic instalment is written below for $t = 1, \ldots, T$.

$$I_t = \begin{cases} I^*, & \omega_t \geq I^* \\ \omega_t I^*/\overline{\omega}, & 0 < \omega_t < I^* \\ 0 & \omega_t \leq 0 \end{cases}$$

If the trader's daily profit is larger than basic instalment I^*, then the trader needs to pay I^* in full. If not, which means she has incurred a loss, then either she pays a portion of I^* or does not pay anything at all. This portion is the ratio between the daily profit and the average value of the trader's daily profit that is calculated before the scheme begins. When the trader does not pay the basic instalment, it will be placed as a debt, accumulated in H_t. At the next occasion when the trader is capable to pay, the debt will be paid as payable debt $C_t, t = 2,3, \ldots, T$ as in Eq. (2). This payable debt can be a full payment of H_t, if she can afford it, or some $k \in N$ sequence of C_t. Below are the formulas for H_t and C_t for $t = 2,3, \ldots, T$.

$$H_1 = I^* - I_1\ ,$$

$$C_t = \begin{cases} H_{t-1}, & \omega_t - I^* - H_{t-1} \geq 0, \\ H_{t-1}/k, & H_{t-1}/k < \omega_t - I^* < H_{t-1}, \\ 0 & \omega_t - I^* \leq H_{t-1}/k, \end{cases}$$

$$H_t = tI^* - \sum_{i=1}^{t} I_i - \sum_{j=2}^{t} C_j$$

Now we calculate the profit sharing in case the trader gains a good profit. The calculation of the profit sharing with portion $p > 0$ is formulated in the following equation.

$$B_t(p) = \begin{cases} p(\omega_t - I_t - C_t), & \text{if } \omega_t - I_t - C_t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Here we determine a good profit by $(\omega_t - I_t - C_t) > 0$. Having deducted the basic instalment I_t and the payment of debt C_t , the excess profit will be shared with the investor in a portion of *p*. Calculating optimal value for p is a question of choosing the objective function to be used. The trader will have a customized value of *p*, depending on her trading profit. In the next section, we explain the transformation of the model the new form in the framework of fuzzy sets.

3.3 Mathematical Modelling in Fuzzy Sets

Firstly we define fuzzy numbers as follows. Variables and parameters in fuzzy numbers are written using upper bar $(\bar{\cdot})$. Firstly, we define the set that will be the support of the fuzzy number.

$$\omega_x = \{x \in \mathcal{R} \mid x = \omega(t), t = 1, \dots, T\}$$

$$\bar{\omega} = \left[\omega^L = \min \omega_x, \omega^C = \tfrac{1}{2}(\omega^L + \omega^R), \omega^R = \max \omega_x\right].$$

Let $\bar{\omega}$ be the profit of daily trading of the trader for a particular period. In defining data in section 4, there are 4 (four) fuzzy numbers that represent 4 (four) conditions of the trader; loss, no-profit, small profit and big profit.

Let $\bar{I}$ be the instalment for paying the principal, $\bar{B}(p)$ be the amount of the profit sharing which contains the profit sharing portion *p*, and $\bar{C}$ be the payable debt. The values of these fuzzy variables are defined by assigning the values from other variables, if needed, point by point. The same process occurs in determining other fuzzy numbers $\bar{I}, \bar{C}, \bar{B}(p)$ that are derived from generally these following sets respectively. However, the specific values defining relationship between 2 (two) fuzzy numbers will be defined in (3), (4), (6) and (7).

$$I_y = \{y \in \mathcal{R} \mid 0 \le y \le I^*\}.$$

$$C_v = \{v \in \mathcal{R} \mid 0 \le v < T \times I^*\}.$$

$$B_w(p) = \{w \in \mathcal{R} \mid 0 \le w \le p \times \max \omega(t), t = 1, \dots, T\}.$$

Let $\bar{\omega}_x$, $\bar{I}_y$, $\bar{C}_v$ and $\bar{B}_w$ respectively be symbols for any values $x \in \bar{\omega}, y \in \bar{I}$, $v \in \bar{C}$ and $w \in \bar{B}$. The total amount of payment $\bar{S}(p)$ on portion-*p* for one period is consisting of the basic instalment and some portion of profit sharing.

$$S_z(\mathrm{p}) = \left\{ \begin{array}{l} z \in \mathcal{R} | \, z = \bar{I}_y + \bar{B}_w(p) + \bar{C}_v, \\ \text{for all } y \in \bar{I}, v \in \bar{C} \text{ and } w \in \bar{B} \end{array} \right\}. \tag{3}$$

The values of instalment $\bar{I}_y$ is defined as follows.

$$\bar{I}_y = \begin{cases} I^*, & \bar{\omega}_x > I^*, \\ \bar{\omega}_x, & 0 < \bar{\omega}_x < \bar{I}^*, \\ 0, & \bar{\omega}_x \le 0\,. \end{cases} \tag{4}$$

The loss-sharing will reflect on the values of all three variables; $\bar{I}, \bar{B}(p)$ and $\bar{C}$. When the trader suffers a loss, she is exempt from paying $\bar{B}(p)$ and $\bar{I}$. However, she still has to pay only the basic instalment I^* later which is accumulated in the debt $H(t)$. Here $H(t)$ is a crisp number function depends on time-*t*.

$$H(t) = t\, I^* - \textstyle\sum_{i=1}^{t} \bar{I}_{y_i} - \sum_{j=2}^{t} \bar{C}_{v_j}\,, t = 2,3, \dots, T. \tag{5}$$

$$\bar{C}_v = \begin{cases} H(t-1), & \bar{\omega}_x - I^* \ge H(t-1), \\ \bar{\omega}_x - I^*, & 0 < \bar{\omega}_x - I^* < H(t-1), \\ 0, & \bar{\omega}_x - I^* \le 0, \end{cases} \tag{6}$$

where $H(1) = I^* - \bar{I}_y$. For a value *p*>0, and for all values of x, y and v, we define $\bar{B}_w$ as follows.

$$\bar{B}_w = \begin{cases} p\left(\bar{\omega}_x - \bar{I}_y - \bar{C}_v\right), & p\left(\bar{\omega}_x - \bar{I}_y - \bar{C}_v\right) > 0, \\ 0, & p\left(\bar{\omega}_x - \bar{I}_y - \bar{C}_v\right) \leq 0. \end{cases} \quad (7)$$

The variables to be used to measure the optimality of the model are the rate of return $\overline{r_s}(p)$ and the portion of take-home incomes of the trader $\overline{d_q}(p)$. We also need the related crisp numbers obtained from the usurer model, the rate of return r_u and the portion of take-home incomes of the trader d_u. The numbers r_u and d_u respectively will be used for the upper and lower bounds for the PLS model.

$$\bar{r}_s(p) = \left\{ s \in \mathcal{R} \mid s = \frac{1}{A}\left(-A + \bar{S}_z(p)\right), \text{for all } z \in \bar{S}(p) \right\}, \quad (8)$$

$$\bar{d}_q(p) = \left\{ \begin{matrix} q \in \mathcal{R} \mid q = \max\{\bar{\omega}_x - \bar{S}_z(p), 0\} \ - H(T), \\ \text{for all } x \in \bar{\omega}, z \in \bar{S}(p) \end{matrix} \right\}. \quad (9)$$

The value of the feasible portion p of the profit share will be the solution of the optimization problems as follows:

$$\max_p F(p) = \frac{\gamma}{K_1^2}\left(\overline{r_s}(p) - r_{BI}\right)^2 + \frac{(1-\gamma)}{K_2^2}\left(\bar{d}_q(p) - \ d_u\right)^2 \quad (10)$$

Constraints:

$$r_{BI} < \overline{r_s}(p) < \overline{r_u} \quad (11)$$

$$\bar{d}_q(p) > \overline{d_u} \quad (12)$$

for all $s \in \bar{r}(p)$, $\in \bar{d}(p)$, $0 < \gamma < 1$, and $K_1 = \max_p(\overline{r_s}(p) - r_{BI})$, $K_2 = \max_p\left(\bar{d}_q(p) - \ d_u\right)$. The lower bound r_{BI} is the daily interest rate of Bank of Indonesia that we set $r_{BI} = \frac{7\%}{252} = 0.0002778$. We make a model of usurer scheme which is a conventional loan by usurer with a high interest rate, 30% for period T. The latter model, owning the return rate r_u and the portion of take-home income d_u , is used as the restriction of our profit-loss sharing model that prohibited being satisfied.

The objective function (10) is defined to be dimensionless so it has maximum value 1 (one). If γ is small, the second part in the right hand side of equation (10) is dominated, so value of $\bar{d}_q(p)$ is far enough from d_u, which means the trader is better off than the investor. On the other hand, if γ is approaching 1, the investor is better off than the trader.

IV. RESULTS AND ANALYSIS

Data used to implement the model is obtained from traders of a local traditional market in Bandung for T=52. All data from traders is clustered into 4 clusters using Fuzzy C-Means method, and the result is presented in Figure 1. The daily profit of traders in Indonesian Rupiah are defined in 4 (four) the triangular form of fuzzy numbers as follows:

$\overline{\omega}^1 = [-3{,}033{,}277, -771{,}551, -45{,}100]$,

$\overline{\omega}^2 = [-670, 35, 373{,}577]$,

$\overline{\omega}^3 = [25{,}281,\ 240{,}040,\ 1{,}78{,}522]$ and

$\overline{\omega}^4 = [475{,}405, \quad 1{,}444{,}853, \quad 4{,}669{,}424]$

Cluster 1 is a negative fuzzy number $\bar{F}$ with $f^R < 0$. Cluster 2 is a fuzzy number $\bar{F}$ with $f^L < 0$, $f^R > 0$. Clusters 3 and 4 are positive fuzzy numbers $\bar{F}$ with $f^L > 0$. See (1.b) for the definition of a triangular form.

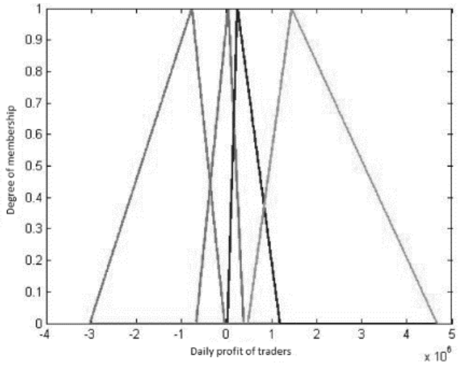

Figure 1.
Clusters of daily profit traders

The procedures of finding the optimal portion *p* is the similar as in [11, 12]. First both $\overline{r_s}(p)$ and $\bar{d}_q(p)$ are computed from (8)-(9) for various values $p = 0.001, 0.002, \dots, 0.3$ Cluster 1 is a negative fuzzy number so it is impossible to obtain $\overline{r_s}(p)$ and $\bar{d}_q(p)$ which satisfy the constraint (11)-(12). Cluster 2 also contains negative part and the constraints are not satisfied. We cannot obtain the results from these two clusters.

The obtained values of $\overline{r_s}(p)$ and $\bar{d}_q(p)$ are approximated so they are in the form of triangular or trapezoidal forms, which show the rate of return per day of the investor and the portion of take-home incomes per day of the trader. For values $\gamma = 0, 0.1, \dots, 1$, the results of the optimization problem (10) – (12) are summarized into 3 (three) fuzzy numbers, which are quite similar. In Figure 2.a graphs (1) and (2), Cluster 3 data respectively yields (0.0031, 0.0498, 0.3943, 0.6028), which are from *p*=0.01 and (0.0035, 0.0552, 0.4398, 0.6631), which is from *p*=0.011. In Figure 2.a graph (3), Cluster 4 data yields (0.0016, 0.0234, 0.2042, 0.3014) which is from p=0.005. Notice that the lowest profit sharing is from Cluster 4 owned by the trader who has higher daily profit.

It can be concluded that the rates of return for Cluster 3 and 4 are about 0.16 – 66.31% per day (depending on the type of the traders), which are higher from the rate or return for deposit from Bank of Indonesia, 0.028 % per day. The return is still below the usurer's rate of return 44% per day. The take-home-income for all traders for both clusters is (0.2367, 0.8247, 0.8247, 0.9729) or about 82 %, which is shown in Figure 2.b. We can see that the benefit for trader is similar for all values of the portion of profit share.

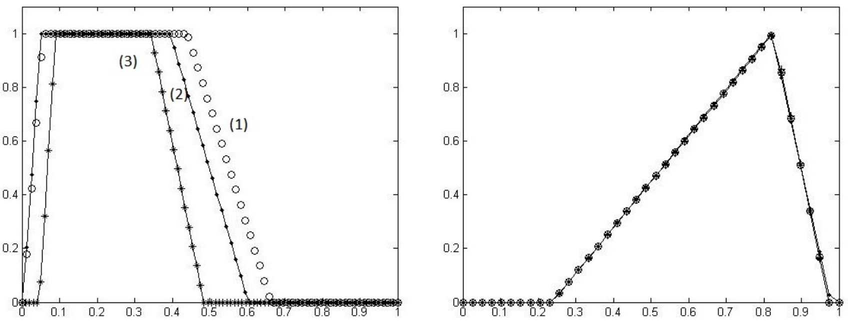

Figure 2.
(a) The optimal rates of return for the investor (left) and
(b) the optimal take-home-income for traders (right)

From previous research in (Sumarti *et al.*, 2014) with the same data but T=60, the obtained values of $\overline{r_s}(p)$ and $\bar{d}_q(p)$ are 0.59% – 0.77% per day and 54.98% – 91.98% per day respectively. Notice these values for the rates of return and the portion of take-home incomes are inside the intervals from the present results.

V. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The implementation of Semi-Fuzzy System in modelling of the profit-loss sharing scheme could simplify the process in finding the values of profit share that give the optimal rate of return for investor and the take-home-income portion for traders. The comparison between the results from (Sumarti *et al.*, 2014) and this paper shows that the fuzzy approach considers more general type of trader and consequently it yields more general results. However, the process of the optimization is still in the operation among crisp numbers for the shake of simplification. For further study, we need the definition of ordering fuzzy numbers in order to transform the original problem fully to the problem of the optimization using fuzzy logic.

Based on the collected data, the optimal profit share is depended on the income of the traders. The higher the income, the lesser the profit share. This results are similar to and supporting the existing results in (Sumarti *et al*., 2014) that used generation data in implementing the model. In the conventional practises where a fixed proportion of profit share is applied, the higher the income of the trader, the larger amount profit given to the investor due to the same portion of profit share for all cases. It is unfair because the results of the hard working effort of the trader will be much consumed by the investor. It can be concluded that our model is appropriate for solving problem in determining the fair portion share in the profit-loss sharing scheme between trader and investor.

5.2 Recommendations

The model of Profit-Loss Sharing Scheme has shown good performance in implementing the *Musharaka* investment between an investor and small-scale traders. It is recommended that the model can be applied into real practices. The period of investment can contain a multiple time intervals which continues from previous and next time intervals. Further study will be needed to see how much this profit share is changing when the capital A and the period of time are varied, and the results could be compared to the results in (Sumarti *et al*., 2015). More challenging open problem theoretically is to define the modelling and the optimization problem in the fuzzy logic approach fully.

REFERENCES

Abu-Joudeh, B. (2012). *Exploring the potential for profit-loss sharing models in Islamic microfinance* (Huntsman Program Honors Senior Thesis). University of Pennsylvania.

Ahmed, H., (2002). Incentive-compatible profit-sharing contracts: A theoretical framework. In M. Iqbal & D. T. Llewellyn (Eds), *Islamic banking and finance: New perspectives on profit-sharing and risk* (pp. 40-54). Northampton: Edward Elgar Publishing.

Amran, A. M., Rahman, R. A., Yusof, S. N. S., & Mohamed, I.S. (2014). The current practice of Islamic microfinance institutions' accounting information system via the implementation of mobile banking. *Procedia-Social and Behavioral Sciences*, *145*, 81-87.

Banerjee, S. & Roy, T. K. (2012). Arithmetic operations on generalized trapezoidal fuzzy number and its applications. *Turkish Journal on Fuzzy Systems*, *3*(1), 16-44.

Beck, T., Demirgüç-Kunt, A., & Merrouche, O. (2013). Islamic vs. conventional banking: Business model, efficiency and stability. *Journal of Banking & Finance*, *37*(2), 433-447.

Chrysafis, K. A., & Papadopoulos, B. K. (2009). On theoretical pricing of options with fuzzy estimators. *Journal of Computational and Applied Mathematics*, *223*, 552-566.

Ensiklopedia Ayat Al-Qur'an & Hadits (Jilid 6 Buku 7). (2016). Jakarta: Widya Cahaya.

Greuning, H., & Iqbal, Z. (2009). Balance sheet analysis: Islamic vs. conventional. *New Horizon*, *170*, 16-17.

Guerra, M. L., Sorini, L., & Stefanini, L. (2011). Option price sensitivities through fuzzy numbers. *Computers & Mathematics with Applications*, *61*(3), 515-526.

Hamza, H. (2016). Does investment deposit return in Islamic banks reflect PLS principle?. *Borsa Istanbul Review*, *16*(1), 2-42.

Haque, T. (2000). New roles for finance in the race to sustainability: The experience of Grameen Bank, Bangladesh. *Corporate Environmental Strategy*, *7*(2), 228-234.

Hasuike, T., Katagiri, H., & Ishii, H. (2009). Portfolio selection problems with random fuzzy variable returns. *Fuzzy Sets and Systems*, *160*(18), 2579-2596.

Imam, P., & Kpodar, K. (2016). Islamic banking: Good for growth?. *Economic Modelling*, *59*, 387-401.

Kumru C. S., & Sarntisart, S. (2016). Banking for those unwilling to bank: Implications of Islamic banking systems. *Economic Modelling*, *54*, 1-12.

Sumarti, N., & Dhanny, Q. (2010). An implementation of investment analysis using fuzzy mathematics. *Proceedings of the Int. Conf. on Industrial and Appl. Math., Indonesia*.

Sumarti, N., & Nadya, P. (2016). A dynamic portfolio of American option using fuzzy binomial method. *Journal of Innovative Technology and Education*, *3*(1), 85-92.

Sumarti, N., & Sukaenah, C. (2010). An Implementation of fuzzy linear system in economics. *Proceedings of the Int. Conf. on Industrial and Appl. Math, Indonesia.*

Sumarti, N., & Wahyudi, N. (2014). Stock and option portfolio using fuzzy logic approach. *AIP Conference Proceedings*, *1589*, 504.

Sumarti, N., Athari, N., & Fadhli, M. R. (2008). An application on multiple-correspondence analysis on the survey for implementation of the profit-loss sharing concept. *Proceedings of International Conference on Mathematics and Natural Sciences, 28-30 October 2008*.

Sumarti, N., Fitriyani, V., & Damayanti, M. (2014). A mathematical model of the Profit-Loss Sharing (PLS) scheme. *Procedia-Social and Behavioral Sciences*, *115*, 131-137.

Sumarti, N., Sidarto, K.A., Syamsuddin, M., Mardiyyah, V. F., & Rizal, A. (2015). Some problems on the making of mathematical modelling of a profit-loss sharing scheme using data simulation. *Journal of Mathematical and Fundamental Sciences*, *47*(1), 1-11.

Yu, T. H. K., Wang, D. H. M., & Chen, S. J. (2006). A fuzzy logic approach to modeling the underground economy in Taiwan. *Physica A: Statistical Mechanics and Its Applications, 362*(2), 471-479.

Yunus, M. (1999). *Banker to the poor: Micro-lending and the battle against world poverty*. New York: Public Affairs.